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SOME OBSERVATIONS ON THE RELATIONSHIP BETWEEN FATIGUE AND INTERNAL FRICTION

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SUMMARY

An investigation has been conducted to determine the internal friction and fatigue strength of commercially pure 1100 aluminum under repeated stressing in torsion at various temperatures and stress levels in an effort to find if there exists any correlation between internal friction and fatigue characteristics. For this investigation a torsional fatigue-testing machine of the resonant type was designed with facilities for measuring internal friction at low stress levels.

Results indicate the existence of a critical temperature at which the fatigue life of the specimens appears to reach a minimum value. The effect of this temperature on the internal friction at various stress levels was quite substantial. In addition, the phenomenon of the recovery of internal friction during brief periods of rest was discussed. The recovery of internal friction was observed to depend upon the stress level and temperature of testing.

INTRODUCTION

While considerable work has been done in order to determine the damping capacity of materials at engineering stress levels and at various temperatures in fatigue stressing, no systematic investigations have been conducted in order to correlate the internal friction at low stress levels with fatigue stress history. Measurements of internal friction at low stress levels have been used in many investigations in order to determine the effects of various kinds of cold-work; these investigations have resulted in some interesting observations. It was consequently felt desirable to initiate a fairly exhaustive experimental investigation in order to correlate fatigue, internal friction, and temperature.

This work is an investigation of the internal friction and fatigue strength of commercially pure 1100 aluminum under repeated stressing in torsion at various temperatures and stress levels. Specifically, the investigation was designed to include: Determination of the critical temperature for the viscous behavior of the grain boundary and its possible relation to the fatigue problem; preliminary investigation of the effects of brief periods of rest on internal friction during progressive stress history at different temperatures; and trends in the variation of internal friction, as measured by the logarithmic decrement δ in free oscillations of small amplitude, as a function of torsional shear stress τ, temperature T, and number of reversals N, in specimens which are stressed cyclically in large-amplitude forced vibrations.

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TEST MATERIAL

Commercially pure 1100 aluminum wires 1/8 inch by 36 inches of the following composition were used in the tests: aluminum, 99.19 percent; iron, 0.49 percent; copper, 0.16 percent; silicon, 0.10 percent; magnesium, 0.03 percent; and manganese, 0.03 percent. This relatively pure aluminum was chosen as the test material because some information was already available from Kê's work (as discussed in ref. 1) and because of the relative simplicity in interpreting the results.

The aluminum wires were straightened before testing by applying a tension of 750 psi and passing a current of 175 amperes at 6 volts through them for a period of 5 minutes, which raised the temperature of the wire to about 750° F. Test specimens 9 inches long were cut from these wires. In the preliminary run of 16 tests, the test specimens which were thus prepared were held by grips in the testing machine and were subjected to an anneal at 750° F for 6 hours. In subsequent tests, thicker ends (as shown in fig. 1) were cast in an induction furnace in order to reduce the extraneous friction at the grips. The second set of specimens was polished with 600A emery paper, annealed at 750° F for 6 hours, and cooled in the furnace to room temperature to remove the effect of prior stress history.

The free length of the test specimen was 6 inches and the diameter was 0.125 inch. The variation was not more than 0.002 inch in diameter and 0.025 inch in length.

TEST EQUIPMENT

The principle of resonance (with an external elastic restraint) was used for producing torsional vibrations. By changing the stiffness of the external restraint, it was found possible to change the operational frequency of forced vibration at resonance. The decay of free oscillations was used for damping measurements and a photocell electronic system was used to facilitate quick measurements.

In preliminary tests it was found that, if an elastic restraint was attached to one end of a two-pole alternating-current motor as shown in figure 2(a), it was possible to excite torsional vibrations in the rotor, provided that the angle θ was not zero. By choosing a proper length for the restraint and a corresponding frequency for the power supply, it was possible to maintain oscillations of substantial amplitude with very little power input. A pair of electromagnets was used to start the test specimen and inertia bar in free vibration. In order to reduce the effect of residual magnetism, these magnets were pulled away from the vicinity of the inertia bar after the free vibrations started. In order to simulate free torsional vibrations only, a torsion pivot support was placed at the end of the inertia bar as shown in figure 2(b). The final setup, based on this principle, is shown schematically in figure 3. The photographs in figure 4 show the testing machine with its associated equipment. The measurement of damping is carried out as shown in figure 5.

ERRORS IN MEASUREMENTS

A detailed discussion of the measurement errors is given in the appendix. The results may be summarized as follows:

At room temperature, for an annealed specimen, there was still a large variation in the measured value of the logarithmic decrement of the free vibration. This variation may be as high as 150 percent from the mean value for different specimens. At higher temperatures the variation was considerably less, going down to values as low as 5 percent.

In the measurement of damping of a specimen, there was an inherent statistical variation of about 10 percent or less in range about the mean value. While looking for significant trends in recovery, this variation was kept in mind.

There was a maximum error of 5.4 percent in the measurement of damping from reading to reading of a specimen between a number of reversals of forced vibration.

The amplitude of forced vibration did not fluctuate more than 4 percent except when the specimen was about to break.

The temperature in the furnace which was calibrated with the thermocouples installed directly on the test specimen (and these related to a thermocouple placed very near the specimen) did not vary beyond ±2° F in the range of 75° to 750° F, and over a period of time the fluctuation was of the same order.

DETERMINATION OF δ

The principle of decay measurements and the equation for internal friction δ are given in figure 5.

The number of cycles in free vibration required for a chosen amplitude decay is hereafter called "the decay cycles." This is the number of cycles required for an amplitude of 1 inch to die down to 0.4 inch on a linear scale at a distance of 130 inches from the test specimen. The the 1-inch amplitude corresponds to a shear strain of 4×10^{-5} radian and the corresponding stress is 140 psi. Substitution of these into the equation for δ given in figure 5 gives the value

$$\delta = \frac{1.833}{n}$$

where n is the number of half cycles measured by the counter. In the text hereafter, the terms "internal friction" and "damping values" are used interchangeably.

Since one of the prerequisites for the measurement of anelastic internal friction is amplitude independence of the measured value of friction, it becomes desirable to determine if the amplitude settings chosen above do give measurements of internal friction independent of amplitude. In order to do this, the damping curves were determined for various initial amplitudes and are shown plotted in figure 6. It can be noticed that up to 500 psi of maximum stress, the damping is independent of stress amplitude. Consequently, it is concluded that the chosen value of stress amplitude for measurement of internal friction is well within the range of amplitude-independent damping.

DETERMINATION OF δ-VERSUS-T RELATION

Since one of the aims of this investigation was to determine the effect of a critical temperature on fatigue, the variation of damping with temperature for a specimen before stressing was determined. Kê's work (refs. 1 and 2) on the grain boundary has shown the existence of a critical temperature for maximum grain-boundary damping. As shown in figure 7 the existence of a critical temperature (450° F) was revealed in this case also. The damping did not go down continuously as it did in Kê's work but showed an increase with temperature after a small drop.

There might be many reasons for this. In the first place, the test material in this work was not so pure as the 99.998 percent pure aluminum that Kê used. Secondly, Kê mentioned that the high-temperature side of the curve was found to be very sensitive to preanneal cold-work, indicating that some other effects connected with cold-work come into the picture. This effect was also attributed by Pearson, Greenough, and Smith, as discussed in reference 3, to slipping at interface corners where the stress is concentrated after relaxation. It is also of some interest to note that the critical temperature was 450° F in this case, as compared with about 536° F in Kê's work. Work of other investigators shows that this difference in the maximum temperature may be attributed to purity and grain size.

In the subsequent tests the variation of δ as a function of stress level τ, temperature T, and number of reversals N was determined. Thus, after annealing the specimen for about 6 hours at 750° F, its temperature was brought down to the testing level and the damping was measured. A particular stress level was chosen and the damping of the specimen was determined after various numbers of reversals N. During the period of damping measurement, which was not more than 15 minutes, the test specimen was not subjected to any high-stress-level forced vibration, and this period could be treated substantially as a rest period. The following stress levels and temperatures were chosen for investigation:

τ, psi.	2,000	3,500	5,000	6,000
T, °F	75	300	450	600

This stage of the testing was conducted in two parts. In part one, it was desired to investigate briefly the effects of rest periods on recovery. Consequently, short periods of rest up to 15 minutes were given each time damping was measured after a period of forced vibration. Sixteen tests were performed under this condition, one for each combination of stress levels and temperatures. In the second part, the period of rest was only that time necessary to measure damping once, which, in general, was about 2 minutes. Under these conditions, three additional tests were performed for each pair of stress levels and temperatures. The second part was considered necessary to determine scatter involved in this work and to confirm the general trends observed in part one.

RECOVERY PHENOMENA

It has been suggested in previous work that a period of rest after a specimen has been subjected to sustained vibration for some time would contribute to a decrease in the damping capacity of the test specimen. This was found to be true in these tests. However, it was also found that the recovery was dependent on the stress level of the forced vibrations and the temperature and that under certain conditions there occurred what may be called negative recovery, in which case the damping increased with time.

In order to analyze and present the preliminary results of these phenomena, the following procedure was adopted.

The damping of a specimen before stressing was determined at a chosen temperature at least six times. In general, this was found to have statistical variations, fluctuating about a mean value. The quotient of the largest of the decay cycles over the smallest in these six readings was taken as an indication of this inherent random variation. The decay cycles were also determined at successive intervals during a period of brief rest after a period of sustained vibrations. The quotient, now defined as the recovery factor, was again determined. This recovery factor was considered significant if it was larger than the corresponding quotient for the specimen before stressing. In addition, it was considered significant only if there was noticed either a monotonic increase or a decrease in a set of readings. Thus, for example, at 2,000 psi and 75° F, a specimen before stressing gave the following readings for decay cycles: 669, 651, 649, 649, 652, and 651, with an average of 654 and a quotient of 669/649 = 1.03. The same specimen after being stressed for 46,500 cycles at ±2,000 psi gave the following successive decay-cycle readings in a period of roughly 15 minutes: 451, 457, 478, 485, 492, and 520. This set shows a clearly monotonic increase in the cycles and the recovery factor is 520/451 = 1.15. Since there was a monotonic change in the decay cycles and since the recovery factor 1.15 was larger than the quotient 1.03, this was considered to be a case of significant recovery. In cases where monotonic decrease was noticed, it is apparent that it would give a recovery factor of value less than 1 and, for obvious reasons, this was designated negative recovery. In order to represent this system graphically, cases where no significant recovery was noticed were assigned a recovery factor of 1.

It must be mentioned that there was usually a time lapse of about 45 seconds before the first reading could be taken. Consequently, if the major part of the recovery took place within this period, it was not observable by this method. Nevertheless, these results are presented since, in many cases, the tests show detectable recovery over a period considerably larger than 45 seconds.

In general, it has been found that the recovery factors so determined varied in a random manner as a function of fatigue stress cycles. Consequently, in the absence of further data at this stage it is considered desirable to work with average recovery factors only. This average is the arithmetic mean of the recovery factors determined roughly 16 times in about 30×10^6 cycles; the recovery factors were determined at shorter intervals at the beginning of fatigue stressing. As an example of the results obtained in these tests, typical tables (tables 1(a) and 1(b)) are included for the conditions of 2,000 psi and 75° F and 2,000 psi and 300° F. The results of this investigation are given in figure 8. Where recovery was observed, it tended to increase at its maximum rate within the first few minutes of a rest period. Subsequent periods of rest gave rise to a more gradual increase in recovery. At higher temperatures, the recovery rate appeared to reach an asymptote within a short time (approximately 5 minutes at 450° F and even less at higher temperatures), and, at room temperature, spot testing showed recovery still taking place at times up to 15 hours. The curves in figure 8 indicate some interesting trends. For example, one notices that the room-temperature (75° F) recovery factor increased with the increase of stress, at first rather rapidly and then more gradually. At 300° F the slope of recovery factor versus stress is much steeper. No explanation is offered for the negative recovery factor which occurs at 2,000 psi at 300° F. At 450° F the specimens start with a negative recovery factor and, with an increase of stress, the curve reaches and merges with the curve for recovery factor 1. At 600° F the curve is identical with the line for recovery factor 1.

RESULTS AND DISCUSSION OF TRENDS IN FATIGUE-DAMPING RELATIONS

The results of fatigue and damping were obtained in two stages. Part one concerns the damping values obtained while investigating trends in recovery. It was noticed at the end of this stage of the work that a substantial decrement could be obtained in the initial value of the damping by casting the ends of the specimen integrally instead of using split collets to hold it in the testing machines. Part two consisted of 48 tests with integrally cast specimens, three tests for each pair of test conditions. Using specimens with grips was not considered to have affected the recovery relations observed insofar as the qualitative picture presented by them is concerned. It may also be mentioned that casting the specimen ends integrally affected only four tests, namely, those at 75° F for stresses of 2,000, 3,500, 5,000, and 6,000 psi, the reason being that at higher temperatures the damping of the specimen itself is, in general, of a higher order than that which the specimen may experience at the split collets. While analyzing the fatigue-damping relations and obtaining mean curves, the damping results obtained in these four tests were neglected for reasons stated above.

A typical set of results in this series of tests is presented in table 2. A summary of the average values of damping is given in table 3. A summary of life to failure of the test specimens is noted in table 4.

The results are presented in the form of curves in figures 9 to 12. In order to present the change in the value of damping from an annealed condition to that of the first measurement, the value of the damping before the specimen is stressed is represented on the y-axis and joined by a straight line to the first measurement in the curves in figures 9(a) to (d). Since the scale on the cycles axis starts at 10^4, the slope of the damping curves in this region is fictitious; the only significant item in this region is the nature of the slope but not the magnitude, which is distorted. The curves depicting the relation between fatigue and damping at the different stress levels and temperatures were drawn on the basis of the arithmetic mean of the damping values at 0, 0.5×10^5, 10^5, 10^6, 5×10^6, 10^7, and 3×10^7. Detailed variation of the damping apart from these mean values was not considered significant, and no importance was attached to it.

Stress Level, 2,000 Psi

The following general features in the variation of damping at a stress level of 2,000 psi are apparent in figure 9(a). (In what follows, the term "initial range" is used to define the range of cycles from 0 to 0.5×10^5.)

At 75° F a substantial change in the damping took place in the initial range and from there on there was a gradual but slow increase in damping. At 300° F the change in the initial range was not so great as it was at 75° F, but one would still notice a general trend toward an increase in damping with an increase in number of cycles. At 450° F there was a radical change, with a decrease in damping with an increase in number of cycles, the slope in the initial range being definitely negative. At 600° F the damping again increased with an increase in number of cycles in the initial range, but, as the fatigue stressing continued, there was a tendency towards a decrease in the damping. There were no failures at this stress level at any of the testing temperatures. It appears that the scatter at 75° F was rather large as compared with that at the higher temperatures. While this is generally true, the logarithmic plotting of damping seems to enhance the actual scatter.

Stress Level, 3,500 Psi

At 75° F, for a stress level of 3,500 psi, a substantial change in the damping took place in the initial range, and then there was a gradual increase in damping similar to that at 2,000 psi but larger in magnitude (fig. 9(b)). At 300° F the change in the initial range was not so great as it was at 75° F, but one would notice an increase in damping with an increase in number of cycles, with the magnitude of the increase being much larger than that at 2,000 psi.

At 450° F one again would notice the characteristic negative slope, as in the case of 2,000 psi, but it is slightly less steep. At 600° F, while the damping increased in the initial range, there was no particular increase in damping with stress history beyond 10^5 cycles. At this stress level also, the scatter in damping values at 75° F was rather large. There were no failures at this stress level at any of the testing temperatures.

Stress Level, 5,000 Psi

At 75° F, for a stress level of 5,000 psi, a substantial change in damping took place in the initial range and the damping continued to rise gradually (fig. 9(c)). The rate of increase at 300° F in the initial range was again positive and the subsequent increase was rather negligible. At 450° F one again would notice the characteristic negative slope in the initial range and the continuous decrease of damping until failure of specimens took place. At 600° F the slope in the initial range was again positive, and the damping continued to rise with stress history until failures took place. Although at room temperature there were no failures at this stress level, at higher temperatures all but one of the test specimens failed. The scatter in damping at this stress level was not large, as was the case at 2,000 and 3,500 psi.

Stress Level, 6,000 Psi

At 75° F, for a stress level of 6,000 psi, a substantial change in damping took place in the initial range, as it did at the lower stress levels, but the damping values with further stress history trend downward (fig. 9(d)). At 300° F one notices a positive slope of the damping curve in the initial range; in addition, a rather steep increase in damping with stress history occurs. At 450° F one again notices the negative slope of the damping curve in the initial range and the subsequent continuous decrease in damping with stress history. At 600° F, in the initial range the damping increased with stress history; beyond this range there was a definite tendency toward a decrease in damping with stress history. This is in opposition to the general behavior at lower stress levels at the same temperature.

Discussion

The most characteristic thing observed in all these curves is that, whereas at all other testing temperatures the slope of the damping curves was positive, at 450° F (which was the critical temperature in the δ-versus-T relation), it was negative.

Further observations can be made on the basis of the following simple analysis:

From the mean fatigue-damping curves it is possible to define a δ-average based on the integrated area under a curve drawn on a linear scale. Since all of the specimens did not run until 3×10^7 cycles, the integration of a curve was carried up to a limit which covers all the curves at the same temperature. This corresponded to 10^7 cycles for room temperature and 10^6 cycles for all other temperatures. Thus $N = 10^7$ for $T = 75^\circ$ F and $N = 10^6$ for $T > 75^\circ$ F. The resulting values of δ-average which were considered characteristic of the respective test conditions are given in table 5. The average values of δ' for the specimens before stressing based on the results of 15 tests at each temperature are also given. If the δ-average based on the integration of the mean fatigue-damping curves at each temperature and stress level is divided by the corresponding δ'-average of the specimens before stressing at the same temperature, one obtains a relative variation of internal friction due to fatigue stressing at various stress levels and temperatures.

The variation of this ratio

$$\alpha = \frac{\delta}{\delta'}$$

with temperature and stress level is given in table 5. Thus, for example, at 75° F the average value of δ' of specimens before stressing based on 15 tests was 6.3×10^{-4}. The average value of δ based on the area under the δ-N curve (for 3 tests) after fatigue stressing at 2,000 psi and 75° F was 29×10^{-4}. The ratio $\alpha = \left(29 \times 10^{-4}\right)/\left(6.3 \times 10^{-4}\right) = 4.6$ is a measure of the change in damping of the test specimens at 2,000 psi and 75° F.

The variations of α with stress and temperature are plotted in figure 10. These two sets of curves reveal some extremely interesting trends. Thus, referring to figure 10(a) in which α was plotted against τ for various parametric values of T, one notices that

(1) At 75° F the value of α increased with stress level indicating that the larger the stress level, the larger the increase in damping. The slope $\frac{\partial \alpha}{\partial \tau}$ increased with stress level, thereby indicating that the rate of increase of damping increased with stress level. In passing, it must be mentioned that this internal friction was measured at low stress levels (140 psi) and was not amplitude dependent.

(2) At 300° F, while one would notice the same trends as were observed in (1), the values of α and $\frac{\partial \alpha}{\partial \tau}$ were much smaller, indicating that the relative increase was diminishing with an increase in temperature.

(3) At 450° F the value of α decreased with an increase in stress and was less than 1, indicating a reduction in damping (as compared with the value obtained before the specimen was stressed) after a period of stressing.

(4) At 600° F, α started with a value greater than 1 and decreased to values less than 1 with an increase in stress level.

Referring to figure 10(b), one notices that

(1) The higher the stress, the greater the rate of reduction of α with an increase in temperature below 450° F.

(2) The slope of $\frac{\partial\alpha}{\partial T}$ tends to a minimum around 450° F, indicating that the relative change in internal friction with temperature at all stress levels was a minimum at 450° F.

Further observations on the fatigue-damping relation are as follows:

(1) While the rate of change of α (and, hence, δ) with temperature seems to reach a minimum at 450° F, the change in the absolute value of δ at this temperature was larger than the change at any other temperature, as can be seen from figure 13.

(2) At all stress levels the peak observed in the δ-versus-T relation for a specimen before stressing was removed by an application of fatigue stressing, as can be seen from figure 12.

(3) Figure 11 shows the average fatigue life plotted against temperature. At a stress level of 5,000 psi the curve shows a tendency for minimum fatigue life at 450° F which was the critical temperature for grain-boundary damping. It may be recalled that the points in this plot were based on the average of the logarithmic value of life obtained in four tests at each temperature. Because of a premature failure, the curve for 6,000 psi shows a minimum around 300° F instead of 450° F, as was the case at 5,000 psi. At room temperature one of the specimens at 6,000 psi did not break within 33×10^6 cycles. However, this value was taken as a conservative estimate of the life of this particular specimen. It will be noticed that this procedure does not alter the general picture. No testing was conducted at temperatures higher than 600° F; thus, the nature of the curves beyond 600° F is not known. In order to facilitate quick examination, all the points of failure are plotted in figure 11, and the curves are drawn through the mean logarithmic values. It is emphasized that further testing is necessary to confirm the behavior in the immediate vicinity of the critical temperature.

COMPARISON WITH OTHER INVESTIGATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

While some of the current results have been observed before by other authors (ref. 4) in investigating the effect of cold-work on aluminum (which results, in themselves, could not be explained by any of the existing theories), it is believed that the present work has given rise to some further observations which need explanation. Most important are the relative variation of internal friction as a function of stress history and the effect of the critical temperature of the grain boundary on the fatigue life.

While reporting some work of Read (p. 137 of ref. 5), Zener comments that vibration at the higher stress levels does not affect the internal friction measured at the lower stress levels; that is, internal friction is a single-valued function of stress amplitude. It is not very clear from this statement whether the vibrations at higher stress levels, that Zener considers in his book, are sustained vibrations or only momentary. This seems to be of some consequence since it is the basis of the work presented in this report, that is, the assumption that sustained vibrations at higher stress levels do indeed influence the measurement of internal friction at lower stress levels, these stress levels being far below the range of the amplitude-dependent-hysteresis stress level. The variations noted were not small. There are instances in this work where the change in the value of internal friction of the specimen after testing approached 10 times that of the specimen before stressing.

It should also be remembered that, while investigating the effect of brief periods of rest on recovery of internal frictions, instances were noticed at a stress of 2,000 psi where the recovery factor was consistently negative, that is, a period of rest contributed to an increase in internal friction instead of a decrease. Although it is true that these are the results of tests on single specimens at each of the test conditions, there does not seem to be any reason to disbelieve the results on the basis of the experimental techniques used. The lack of recovery, noticed at higher temperatures, may be due to the fact that, by the time the first reading of the series was taken, there was a lapse of about 30 to 45 seconds; therefore, it is considered possible that the recovery may have already taken place at these temperatures. There is also another problem at these temperatures which is likely to affect the whole work. The minimum recrystallization temperature for the aluminum used in this work is of the order of 300° F; at temperatures higher than this, results noticed may be due to the conflicting action of the fatigue stressing resulting in cold-work and the recrystallization effects of the higher temperatures.

It was assumed that the purity of the test material (99.19) was high enough to neglect the effects of impurities. This may not be quite justified since, for example, it is known that impurities in metals have a tendency to segregate in grain boundaries and that very small amounts of impurities can completely block grain-boundary slip in aluminum. It is also known from Kê's work on grain boundaries that the heats of activation for volume diffusion and relaxation of grain boundaries are roughly the same for pure aluminum. It is, therefore, considered possible that some of the effects noticed may be due to the impurities. Further investigation on this point is needed.

SUMMARY OF RESULTS

From an investigation conducted to determine the internal friction and fatigue strength of commercially pure 1100 aluminum under repeated stressing in torsion at various temperatures and stress levels, the following results were obtained:

(1) For specimens before stressing there existed a critical temperature (450° F) at which the damping of the test material reached a maximum, decreasing at both higher and lower temperatures.

(2) The slope of the curve of internal damping versus number of cycles was positive in the range of 0 to 50,000 cycles at all test temperatures except 450° F. At 450° F, the slope was consistently negative at all stress levels indicating a decrease in internal friction with an increase in stress history.

(3) Based on the failures observed at 5,000 psi and 6,000 psi, the critical temperature of the grain-boundary internal friction (approximately 450° F) appears to have an important bearing on the fatigue life of a specimen considering the average of four test specimens at each stress level and temperature, the fatigue life appears to reach a minimum at 450° F and tends to increase at temperatures both above and below this value.

(4) A tendency toward recovery was observed at some stress levels and temperatures. Not enough data are available to define accurately the recovery rate as a function of stress and temperature, but certain trends have been established. Where recovery was observed, it tended to increase at its maximum rate within the first few minutes of a rest period. Subsequent periods of rest gave rise to a more gradual increase in recovery. At higher temperatures, the recovery rate appeared to reach an asymptote within a short time (approximately 5 minutes at 450° F and even less at higher temperatures) and, at room temperature, spot testing showed recovery still taking place at times up to 15 hours.

California Institute of Technology,
Pasadena, California, June 1, 1955.

APPENDIX

MEASURING TECHNIQUES AND ESTIMATION OF ERRORS

The validity of correlating the damping phenomena with the fatigue properties largely stems from the fact that both of them have the same origin. Since damping is an indirect measure of certain things happening in the material, the changing damping values characterize, at any particular moment, the physical structure at that instant and under those conditions only. Consequently, it is imperative that the conditions under which the damping of a member is being measured be the conditions under which the other physical properties (such as fatigue) are also being measured.

There are many ways in which energy can be dissipated in a system under observation. These can be broadly divided into the following two categories:

(1) External influences.

(2) Internal dissipation such as internal friction due to atomic movements in the grain boundary, slipbands, thermoelastic and magnetoelastic phenomena, and so forth. This class of dissipative mechanisms is, in general, difficult to isolate into its various parts except possibly under some special circumstances. It is, however, possible to isolate the mechanisms to a certain extent in this work. Of all the internal dissipative mechanisms that may arise in this work, only two are of some consequence. They are the slipbands and the grain boundaries. In torsional oscillations, thermoelastic phenomena do not enter because of the fact that no temperature gradients are involved in problems of shear stress and, since aluminum is nonferrous, internal friction due to magnetic hysteresis is of no importance. The major dissipative mechanism, therefore, is the internal friction associated with interfaces such as slipbands and grain boundaries.

In this work there are five places where extraneous damping due to external influences can occur. They are as follows:

(1) The bearings at the ends of the vibrator

(2) The grip between the collets and the specimen at the ends of the test specimen

(3) The supporting mechanism at the bottom of the inertia bar whether it be flexure pivot, torsion pivot, or any other type of support

(4) Magnetic hysteresis due to possible stray magnetic fields

(5) Aerodynamic damping due to the movement of the inertia bar in the air

Every effort was made to reduce the effect of the above items as much as possible.

The bearings at the ends of the vibrator did not contribute to the damping in free vibration to any extent. This is illustrated by the fact that no change was noticed in the value of δ for a specimen before stressing, beyond small random variations, when the top extension rod was held rigid just below the bearings. Furthermore, in the damping tests, the angular movement at the bearings was of the order of $1/400$ of 1°. Since energy dissipation is proportional to the angular displacement and since the displacement is small, this effect was neglected.

Determination of the friction at the collet grips was more difficult. No estimates or measurements seem to be possible. Before a particular test was started, the screws facing the collets in the extension rods were tightened as much as possible and checked again at the end of the test. Furthermore, whenever suspected, the collets were changed in order to avoid friction losses in that vicinity. It is still possible that a certain amount of damping exists here. The collets were used for the first 16 tests only. In later testing, specimens with integrally cast ends were used. In essence, using collets altered the picture in only four cases, as was mentioned in the main text, since at higher temperatures the damping of the specimen itself was, in general, of a higher order than that experienced by the specimen at the collets. With specimens having integrally cast ends, the problem of friction at the grips did not arise since the relative movement observed between the specimen and the collets was absent in this case.

Maximum friction was observed at the bottom of the inertia bar where the bottom extension rod has to be supported in order to simulate pure torsional vibrations. For reasons already mentioned, torsion pivots were adopted. The energy dissipation in the pivots is of the same nature as, but much smaller than, that of the test specimen, because the amount of material and the order of stress involved were very small.

For starting the free vibrations of the test specimen impulsively, a pair of electromagnets and steel pole pieces on the inertia bar were used. Means were designed whereby the electromagnets can be pushed away from the inertia bar when the inertia bar once starts its free oscillation in order to reduce the effect of the residual magnetism of the electromagnets on the pole pieces.

The aerodynamic damping is negligible since the frequency of vibration in the damping tests is small (4 cps) (ref. 6).

A variation from the mean of as much as 150 percent was detected in the damping of the unstressed specimens at room temperature. This variation in damping was attributed to all of the causes previously mentioned and it is suspected, in addition, that a major part of the damping is inherent in the material even after the annealing process and is considered the result of the unavoidable handling processes involved while mounting the specimen in the machine (ref. 4).

The error mentioned above is the cumulative error in initial damping from specimen to specimen; errors of a different nature exist in measurements of damping of one specimen after various numbers of reversals. For example, it is found in the first stage of testing that, when a specimen is annealed and its damping is measured, the variations are clearly statistical in nature with a variation of about ±35 cycles in 700 cycles at room temperature in a typical case. The variations at higher temperatures are, in general, of a smaller order. When interpreting the results, this inherent random variation is kept in mind and only those variations of behavior which appreciably depart from these inherent variations are taken into account.

Still another type of error exists in the measurements and it has its source at the photocell. If y_1 and y_2 are a pair of adjacent amplitudes in free vibration with positive damping, then the ratio

$$\delta = \log_e \frac{y_1}{y_2}$$

is a measure of the damping of the system. This ratio is commonly called the logarithmic decrement. If the equivalent viscous damping is assumed constant, then

$$\begin{aligned} \delta &= \log_e \frac{y_1}{y_2} \\ &= \log_e \frac{y_2}{y_3} \\ &= \ldots \log_e \frac{y_{n-1}}{y_n} \end{aligned}$$

and, hence,

$$\delta = \frac{1}{n}\left(\log_e y_1 - \log_e y_n\right)$$

where y_1 is the initial amplitude and y_n is the amplitude after n number of cycles.

In the measuring technique used herein, errors can be made in locating y_1 and y_n. Consequently, estimating their effect is worthwhile.

One can put the above equation in the form (K is a constant)

$$n = K \log_e \frac{y}{y_n}$$

Suppose in setting y, the starting amplitude of free vibration, an error Δy is made. The number of cycles would now be different and let it be $n + \Delta n$. Then,

$$\Delta n = k\left(\log_e \frac{y + \Delta y}{y_n} - \log_e \frac{y}{y_n}\right) \approx K \frac{\Delta y}{y}$$

where K is a constant. Therefore, the percent of variation is

$$100 \frac{\Delta n}{n} = 100 \frac{K}{n} \frac{\Delta y}{y}$$

$$= 100 \frac{\Delta y}{y \log_e \frac{y}{y_n}}$$

In a similar manner one can obtain the error in the number of cycles when an error of Δy is made in y_n. This would be the case if the slit in front of the photocell is not located accurately. This amounts to

$$100 \frac{\Delta y_n}{y_n \log_e \frac{y}{y_n}}$$

The maximum possible errors in setting are $\Delta y = \frac{1}{40}$ inch, $\Delta y_n = \frac{1}{100}$ inch, $y = 1$ inch, and $y_n = 0.4$ inch. Therefore, the error in setting y is 2.7 percent, and the error in setting y_n is also 2.7 percent.

Since it is conceivable that errors in y can be committed in the initial zero setting, it is apparent that this has to be added twice. In order to reduce the errors in y_n, the photocell location (and, hence, y_n) was kept constant throughout this series of tests and, consequently, there is a total error of $2 \times 2.7 = 5.4$ percent.

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TABLE 1

RECOVERY DATA FROM TESTS ON SPECIMEN[a] UNDER STRESS OF 2,000 PSI

(a) 75° F

Number of reversals, N	Decay cycles for test						δ, (based on test 1)	Significant recovery factor
	1	2	3	4	5	6		
0	669	651	649	649	652	651	27.8×10^{-4}	1.02
$.0465 \times 10^6$	451	457	478	485	492	520	40.6	1.15
.0930	412	416	420	422	426	428	44.6	1.00
.1860	444	429	414	405	425	425	41.3	.96
.2790	461	462	435	430	428	412	39.8	.90
.5580	559	550	550	563	553	560	32.8	1.00
.8370	474	503	521	521	523	530	38.7	1.12
2.511	496	507	525	536	552	555	36.9	1.12
3.348	571	569	573	568	573	575	32.1	1.00
6.696	568	578	592	602	611	618	32.3	1.09
10.00	584	585	593	589	597	599	31.4	1.00
13.39	608	608	609	619	629	634	30.1	1.04
16.92	586	583	598	608	621	633	31.3	1.09
23.43	624	618	634	643	645	634	29.4	1.04
26.78	639	642	667	674	682	702	28.6	1.10
30.13	640	646	660	670	670	678	28.6	1.08

[a]Specimen did not break.

TABLE 1.- Concluded

RECOVERY DATA FROM TESTS ON SPECIMEN[a] UNDER STRESS OF 2,000 PSI

(b) 300° F

Number of reversals, N	Decay cycles for test						δ, (based on test 1)	Significant recovery factor
	1	2	3	4	5	6		
0	182	182	184	182	181	181	100.5×10^{-4}	1.01
$.0465 \times 10^6$	149	150	150	150	151	150	123.0	1.00
.0930	142	143	142	144	144	147	129.0	1.04
.1860	150	148	145	147	147	149	122.0	.99
.2790	148	151	147	146	142	145	124.0	.98
.5580	157	156	157	156	154	153	117.0	.97
.8370	151	146	147	143	137	136	121.5	.90
1.65	179	179	180	180	178	176	102.5	.98
3.35	190	184	182	182	181	179	96.5	.94
6.69	195	187	185	185	185	185	94.0	.96
10.00	196	193	191	189	186	199	104.0	.95
14.22	187	178	176	174	175	174	98.0	.93
16.92	192	185	181	180	176	175	95.5	.91
23.4	186	182	183	182	183	183	98.5	.98
26.78	170	174	165	159	157	155	108.0	.91
30.13	172	170	169	165	164	164	107.0	.95

[a]Specimen did not break.

TABLE 2

INTERNAL FRICTION FROM TESTS ON SPECIMENS UNDER STRESS OF 6,000 PSI

AT TEMPERATURE OF 75° F

Specimen 1		Specimen 2		Specimen 3 (a)		Specimen 4	
Number of reversals, N	δ	Number of reversals, N	δ	Number of reversals, N	δ	Number of reversals, N	δ
0	24.1×10^{-4}	0	6.44×10^{-4}	0	7.95×10^{-4}	0	8.05×10^{-4}
$.0525 \times 10^6$	83.3	$.053250 \times 10^6$	36.70	$.053250 \times 10^6$	46.70	$.053250 \times 10^6$	40.70
.1050	89.3	.106500	69.00	.106500	47.50	.106500	47.70
.2100	74.2	.213000	71.30	.213000	62.30	.213000	44.00
.3150	75.5	.319500	75.70	.319500	62.00	.319500	39.70
.6300	77.00	.639000	78.00	.639000	58.50	.639000	50.60
.9450	56.0	.958500	72.70	.958500	43.30	.958500	54.40
1.89	61.0	1.278	69.70	1.278	62.50	1.278	50.00
3.78	53.7	1.917	65.00	1.917	60.70	1.917	48.20
5.67	52.7	2.556	54.80	2.556	49.50	2.556	37.30
7.56	49.5	3.834	50.40	3.834	58.70	3.834	42.50
9.45	49.7	5.112	32.00	5.112	51.40	5.112	53.60
9.66	Broke	7.668	59.20	7.668	49.70	7.668	39.30
		10.224	36.40	12.78	54.20	10.224	59.50
		14.697	Broke	15.336	56.90	15.336	49.80
				20.448	49.30	17.573	Broke
				28.116	48.60		
				33.228	48.60		

[a]Specimen did not break.

TABLE 3

AVERAGE VALUES OF δ ARRANGED FOR VARIOUS STRESS LEVELS

Number of reversals, N	δ_{av} at -							
	75° F	300° F	450° F	600° F	75° F	300° F	450° F	600° F
	For 2,000 psi				For 3,500 psi			
0	5.9×10^{-4}	65×10^{-4}	496×10^{-4}	417×10^{-4}	5.4×10^{-4}	67×10^{-4}	509×10^{-4}	392×10^{-4}
$.5 \times 10^6$	24.0	80	468	520	25.0	85	418	428
.1	25.0	91	445	519	15.0	88	404	443
1	28.0	74	373	499	24.0	98	357	441
5	27.0	84	335	497	36.0	88	333	442
10	34.0	99	329	466	46.0	88	315	442
30	32.0	83	324	472	33.0	104	304	455
	For 5,000 psi				For 6,000 psi			
0	6.5×10^{-4}	84×10^{-4}	460×10^{-4}	398×10^{-4}	7.5×10^{-4}	94×10^{-4}	433×10^{-4}	404×10^{-4}
$.5 \times 10^6$	21.0	106	357	447	41.0	114	331	407
.1	36.0	102	332	463	55.0	113	319	386
1	41.0	109	299	471	60.0	178	290	381
5	43.0	---	---	---	46.0	---	---	---
10	44.0	---	---	---	49.0	---	---	---
30	40.0	---	---	---	---	---	---	---

TABLE 4

SUMMARY OF LIFE TO FAILURE OF SPECIMENS

Specimen	Life, cycles, at -			
	75° F	300° F	450° F	600° F
	For 5,000 psi			
1	33.0 × 10^6 →	33.0 × 10^6 →	16.83 × 10^6	7.68 × 10^6
2	33 →	4.9	5.12	19.17
3	33 →	5.8	2.78	5.91
4	33 →	7.1	5.43	6.18
Average life	33 →	12.7	7.54	9.73
Average log N	7.5185	6.9559	6.7786	6.9327
Specimen	For 6,000 psi			
1	9.79 × 10^6	4.69 × 10^6	4.26 × 10^6	3.47 × 10^6
2	14.69	1.04	1.94	4.05
3	33.00	1.59	1.91	4.58
4	17.57	2.66	1.76	2.02
Average life	18.75	2.49	2.47	3.53
Average log N	7.2303	6.3290	6.3608	6.5290

TABLE 5

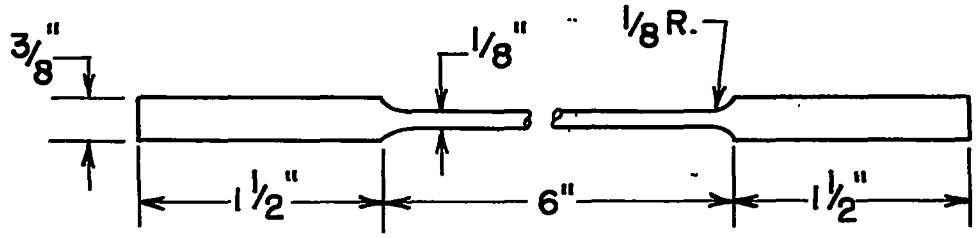
AVERAGE VALUES OF INTERNAL FRICTION AND CORRESPONDING VALUES OF α

FOR VARIOUS STRESSES AND TEMPERATURES

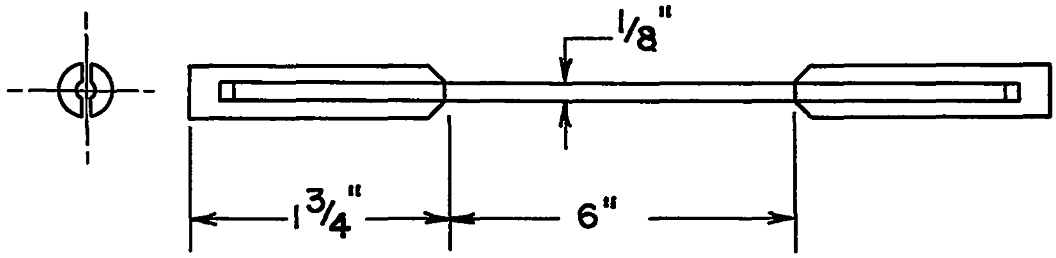
Stress, psi	δ based on area integration of δ-N curve at -			
	75° F	300° F	450° F	600° F
2,000	29.0×10^{-4}	82.0×10^{-4}	414.0×10^{-4}	507.0×10^{-4}
3,500	34	92	386	440
5,000	41	105	322	464
6,000	51	142	309	385
[a] δ'	6.3×10^{-4}	78.0×10^{-4}	475.0×10^{-4}	402.0×10^{-4}
Stress, psi	Corresponding values of α (b)			
2,000	4.6×10^{-4}	1.05×10^{-4}	0.87×10^{-4}	1.27×10^{-4}
3,500	5.38	1.18	.81	1.10
5,000	6.5	1.35	.68	1.16
6,000	8.1	1.82	.65	.96

[a] δ_{av} for annealed specimens.

[b] $\alpha = \frac{\delta}{\delta'}$.

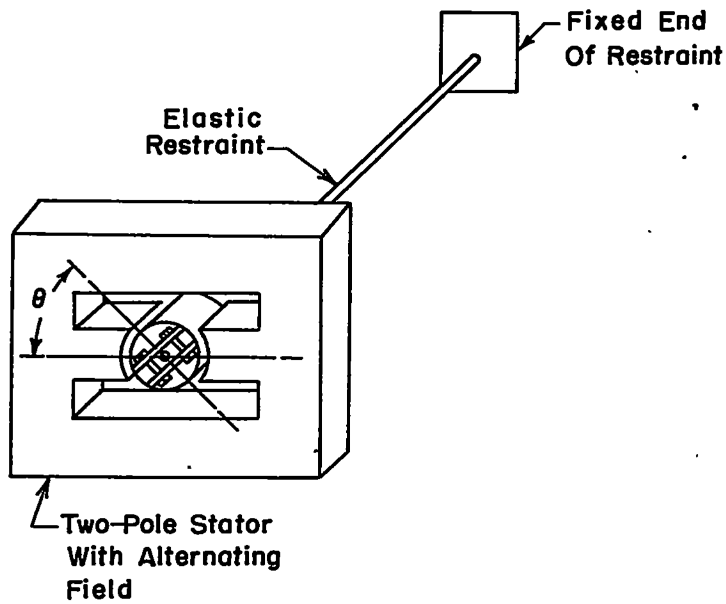


(a) Specimen with integrally cast ends.

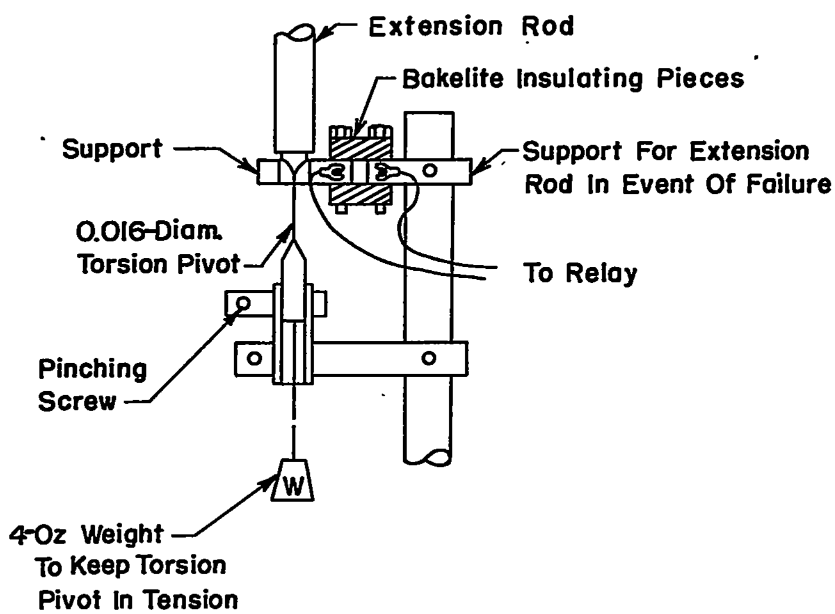


(b) Specimen with end grips.

Figure 1.- Test specimens.



(a) Schematic diagram of vibrator.



(b) Details of torsion pivot.

Figure 2.- Details of test equipment.

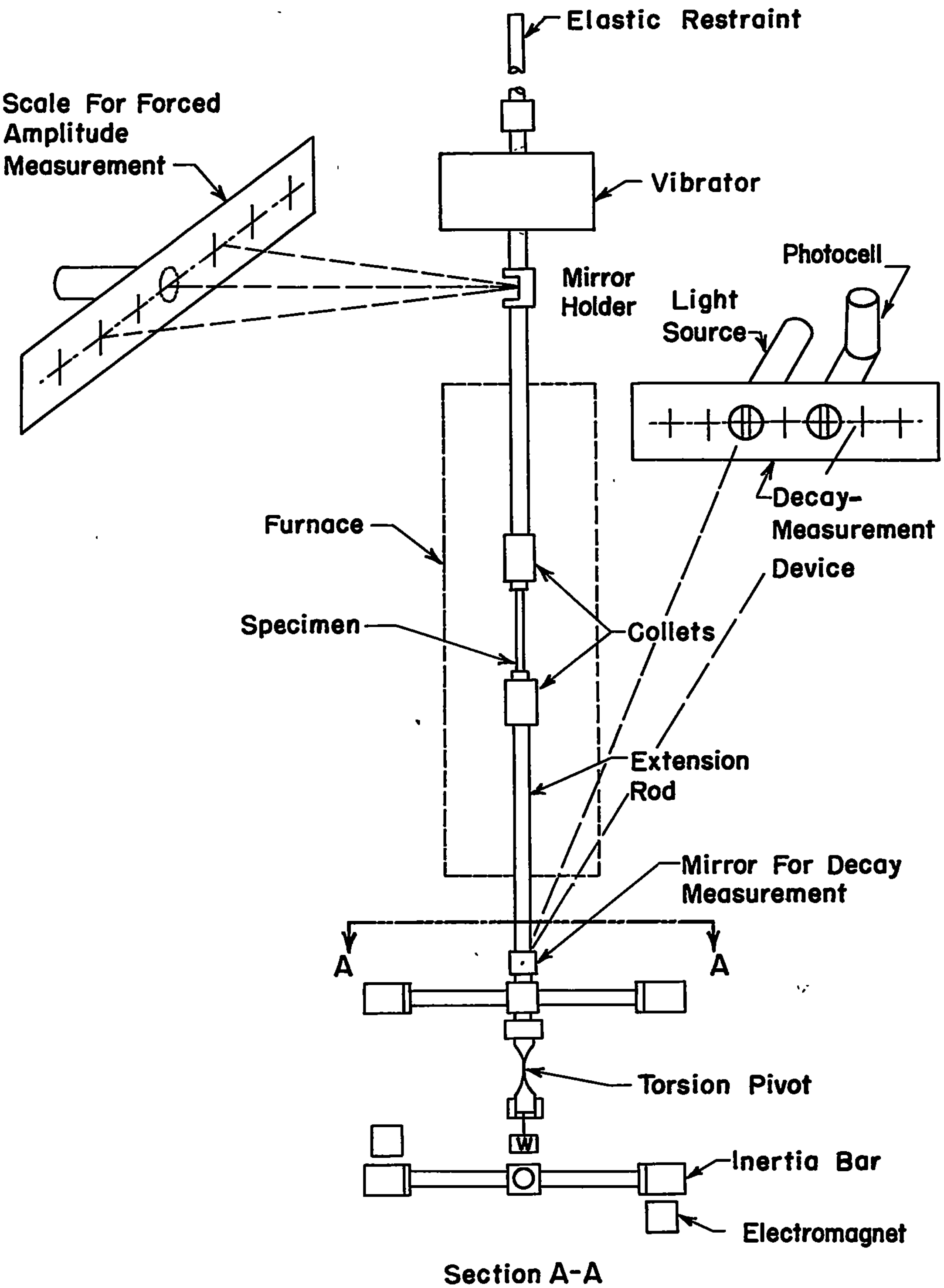
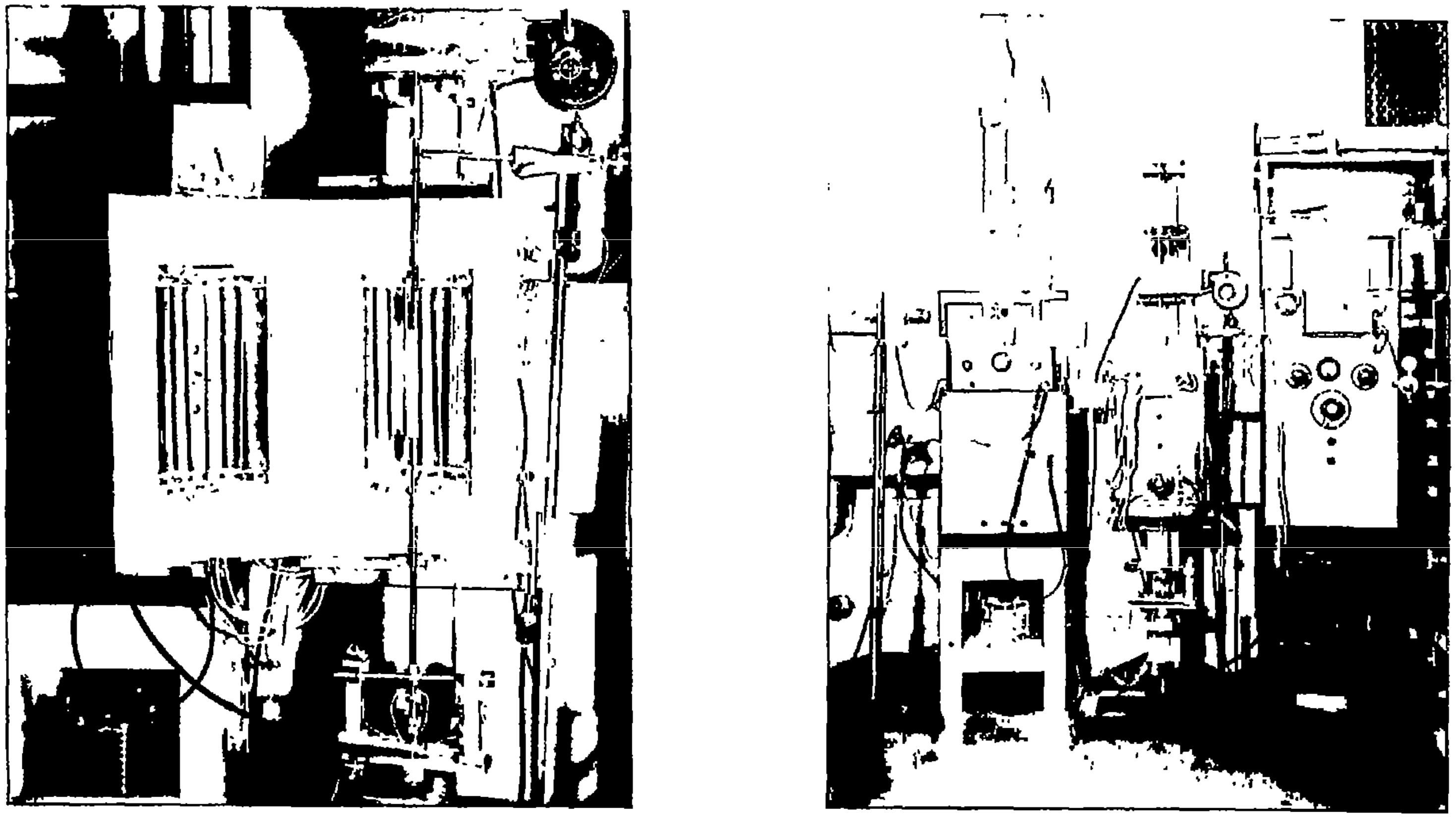


Figure 3.- Schematic diagram of test setup.

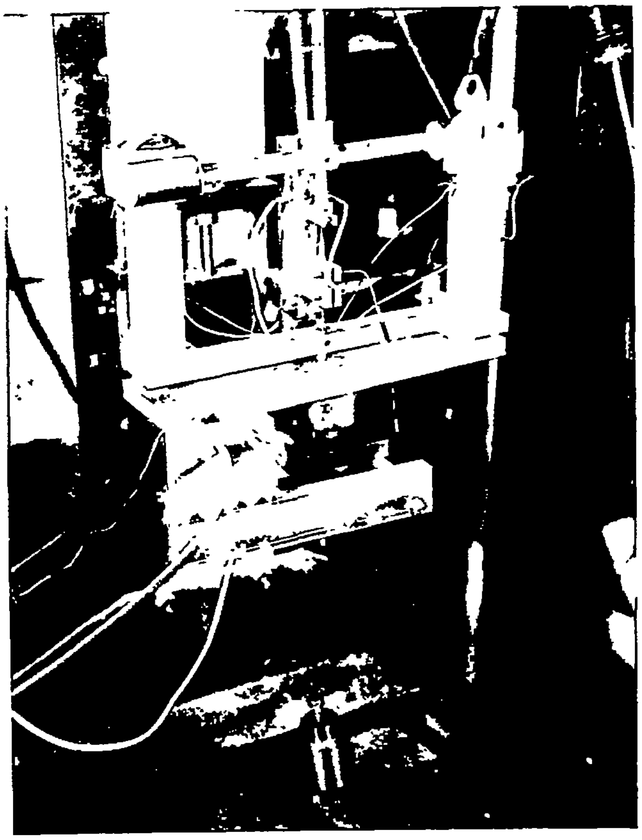


(a) Furnace.

L-93513

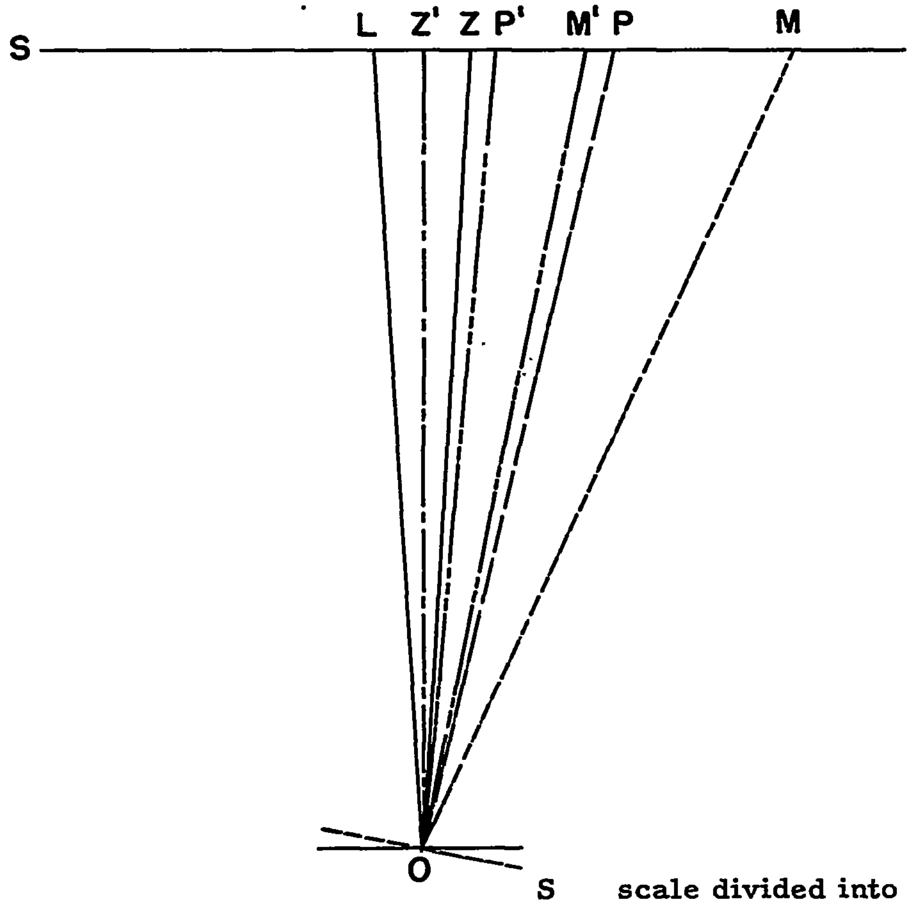
(b) General view.

Figure 4.- Testing machine and associated equipment.



(c) Torsion pendulum. L-93514

Figure 4.- Concluded.



S scale divided into 1/20 inch
LO incident ray
ZO reflected ray with inertia bar in zero position
Z'O bisector of angle LOZ
MO reflected ray with inertia bar attracted by magnets
M'O bisector of angle MOZ
P location of the photocell slit
P'O bisector of the angle POL
n is the number of half cycles

Figure 5.- Principle of decay measurements.

$$\delta = \frac{2}{n}\left(\log_e M'\hat{O}Z' - \log_e P'\hat{O}Z'\right) = \frac{2}{n}\left(\log_e MZ - \log_e PZ\right).$$

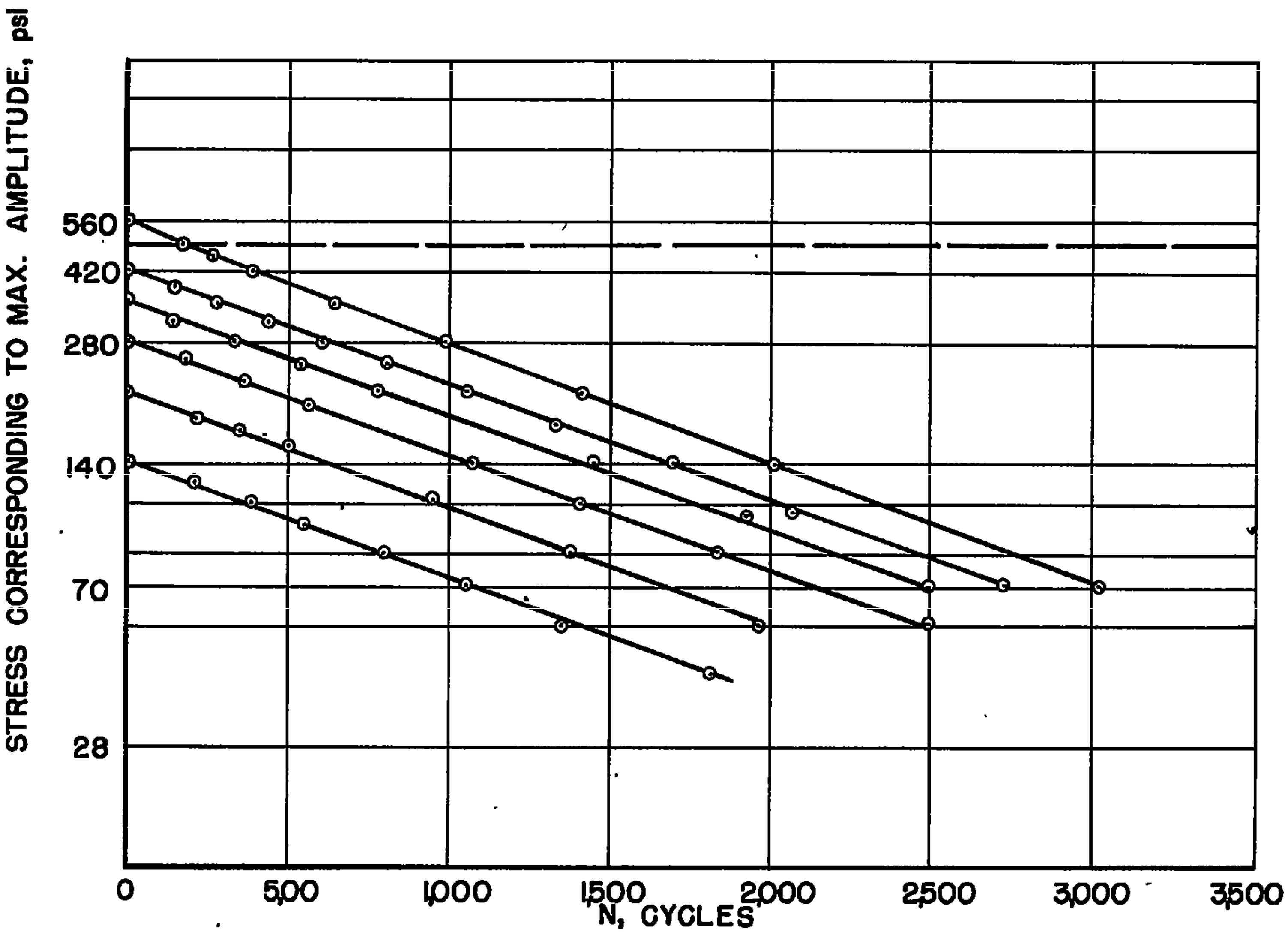


Figure 6.- Damping curves for 1,100 aluminum in torsion.

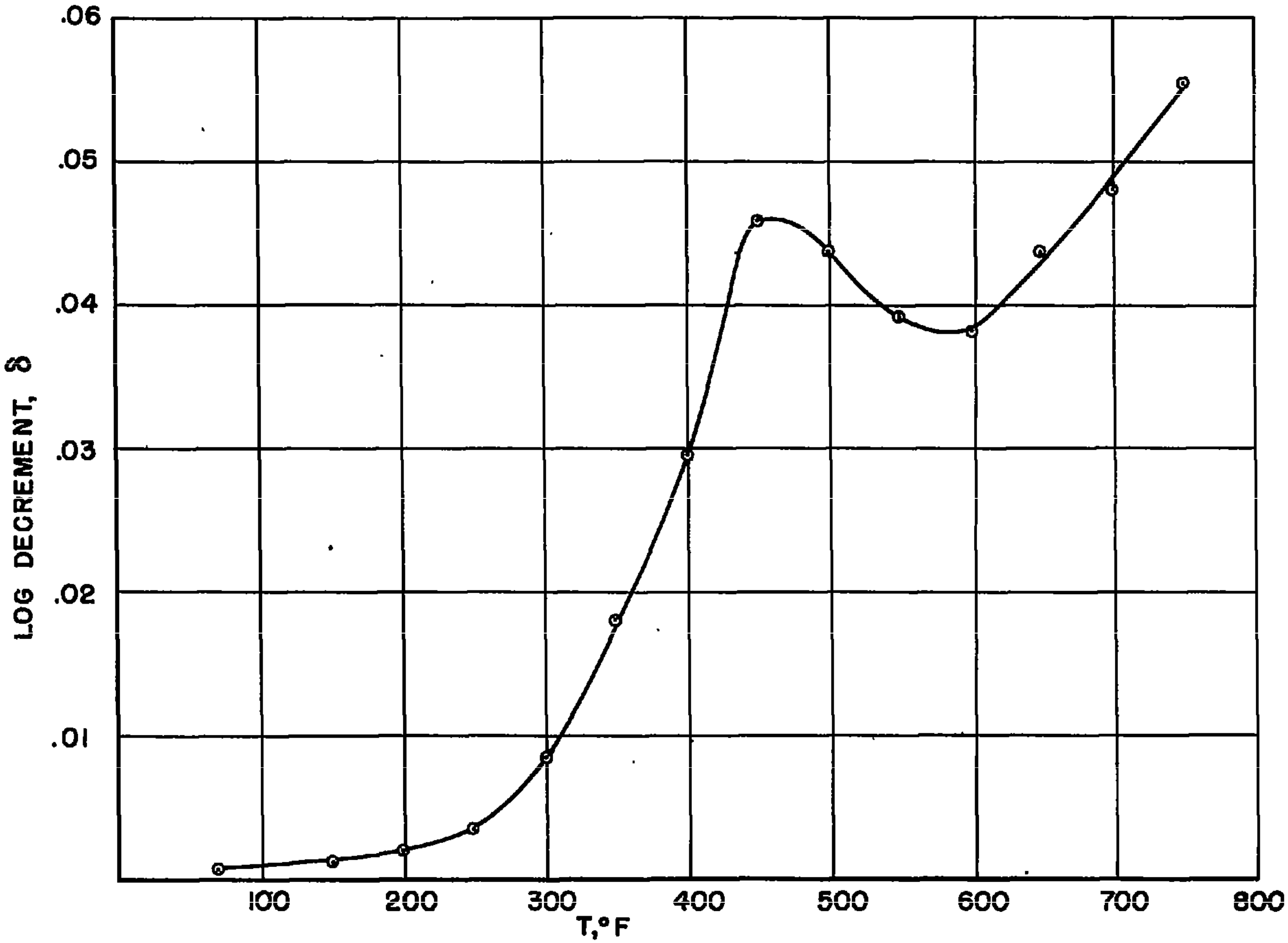
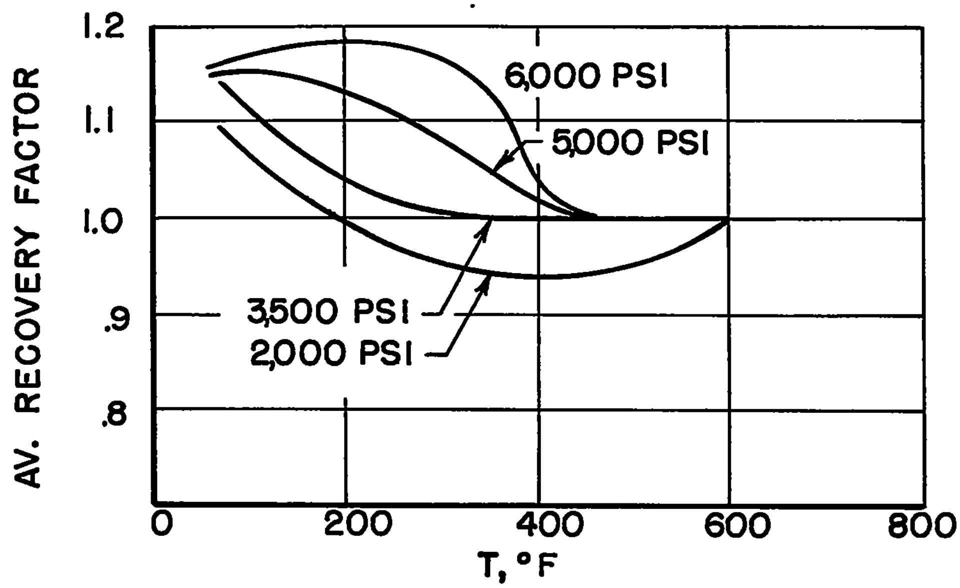
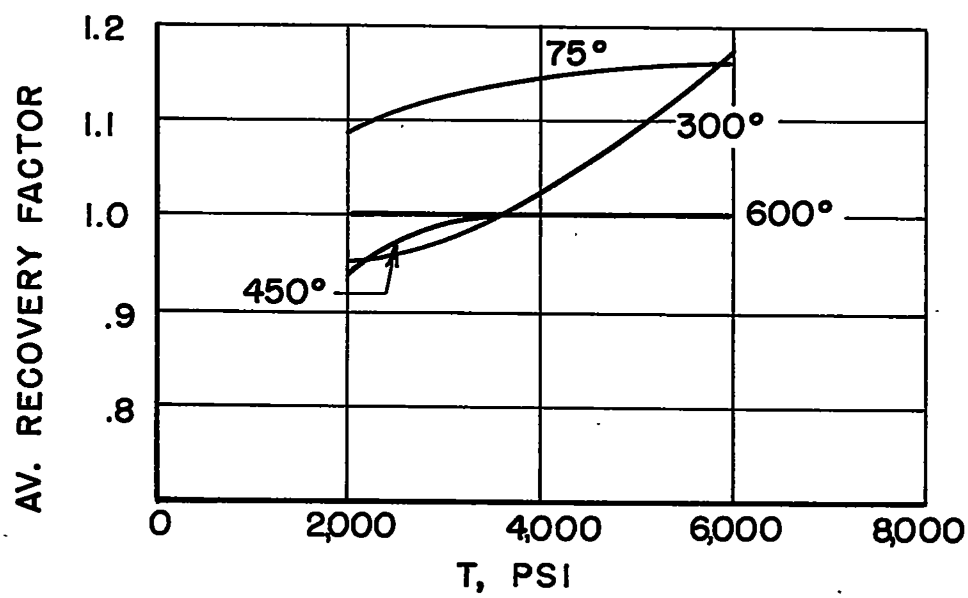


Figure 7.- Variation of internal friction with temperature. Frequency, 4 cps at 75° F.

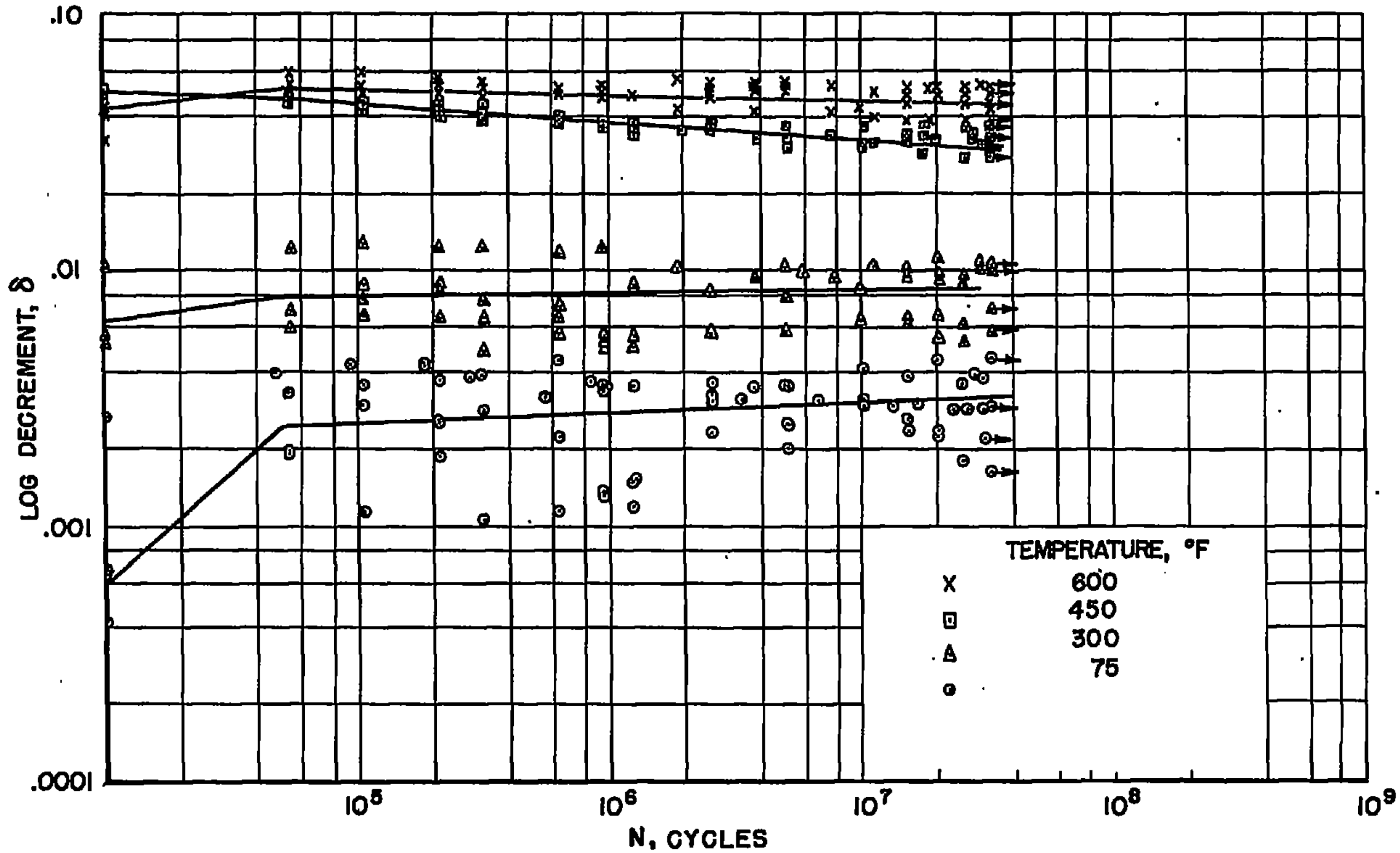


(a) With temperature.



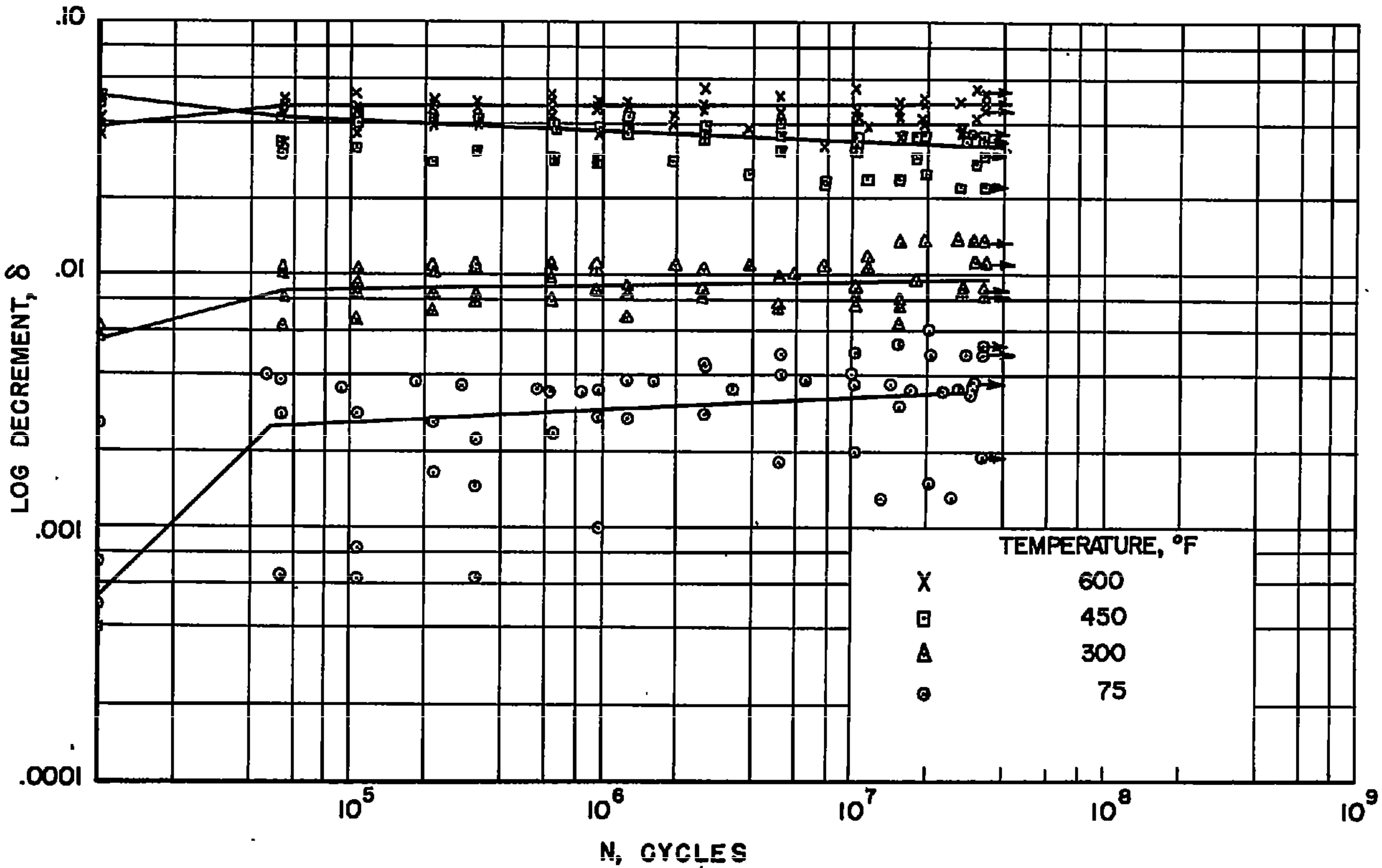
(b) With stress.

Figure 8.- Variation of recovery factor.



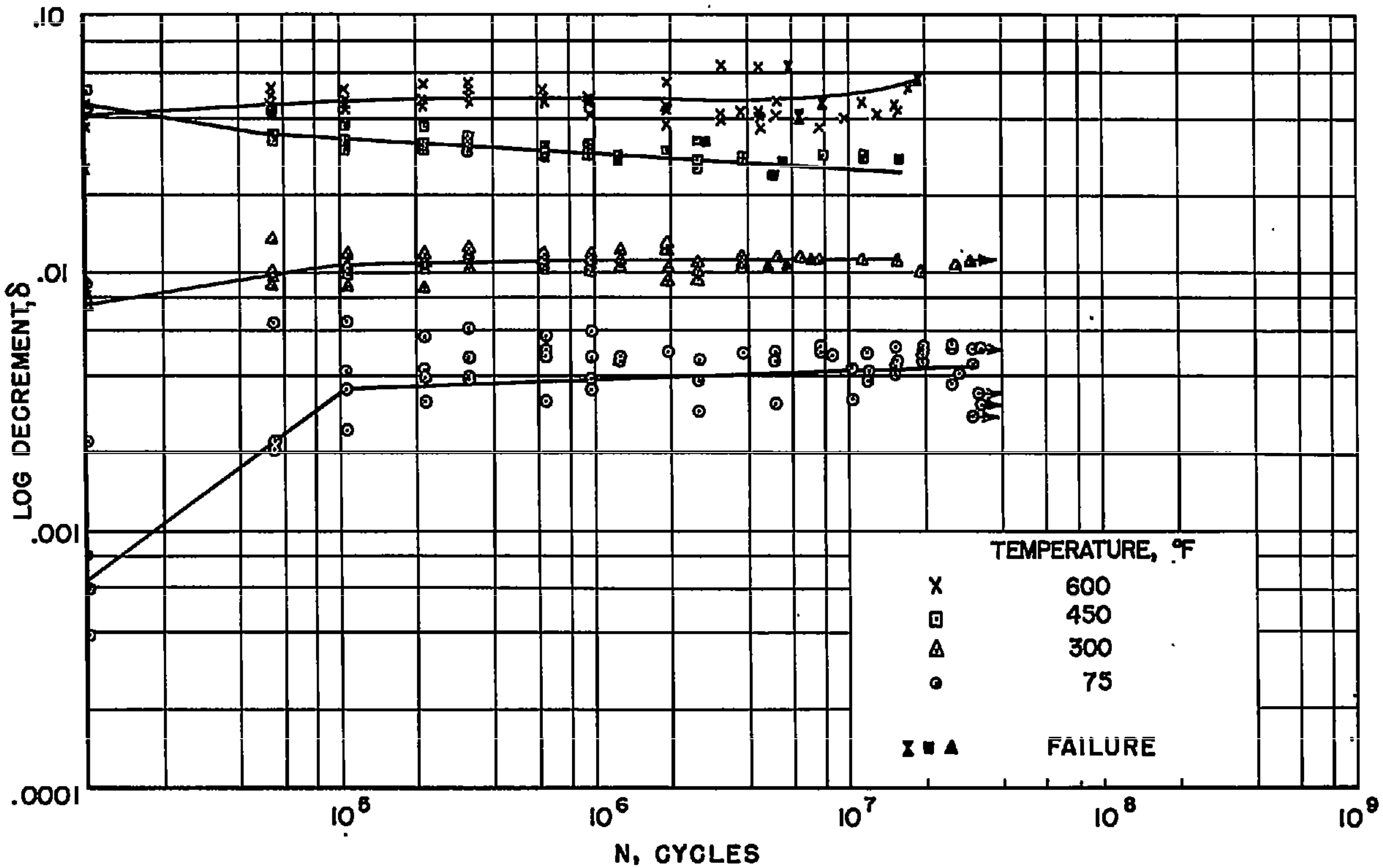
(a) Stress, 2,000 psi.

Figure 9.- Plot of logarithmic decrement δ versus cycles N for various temperatures.



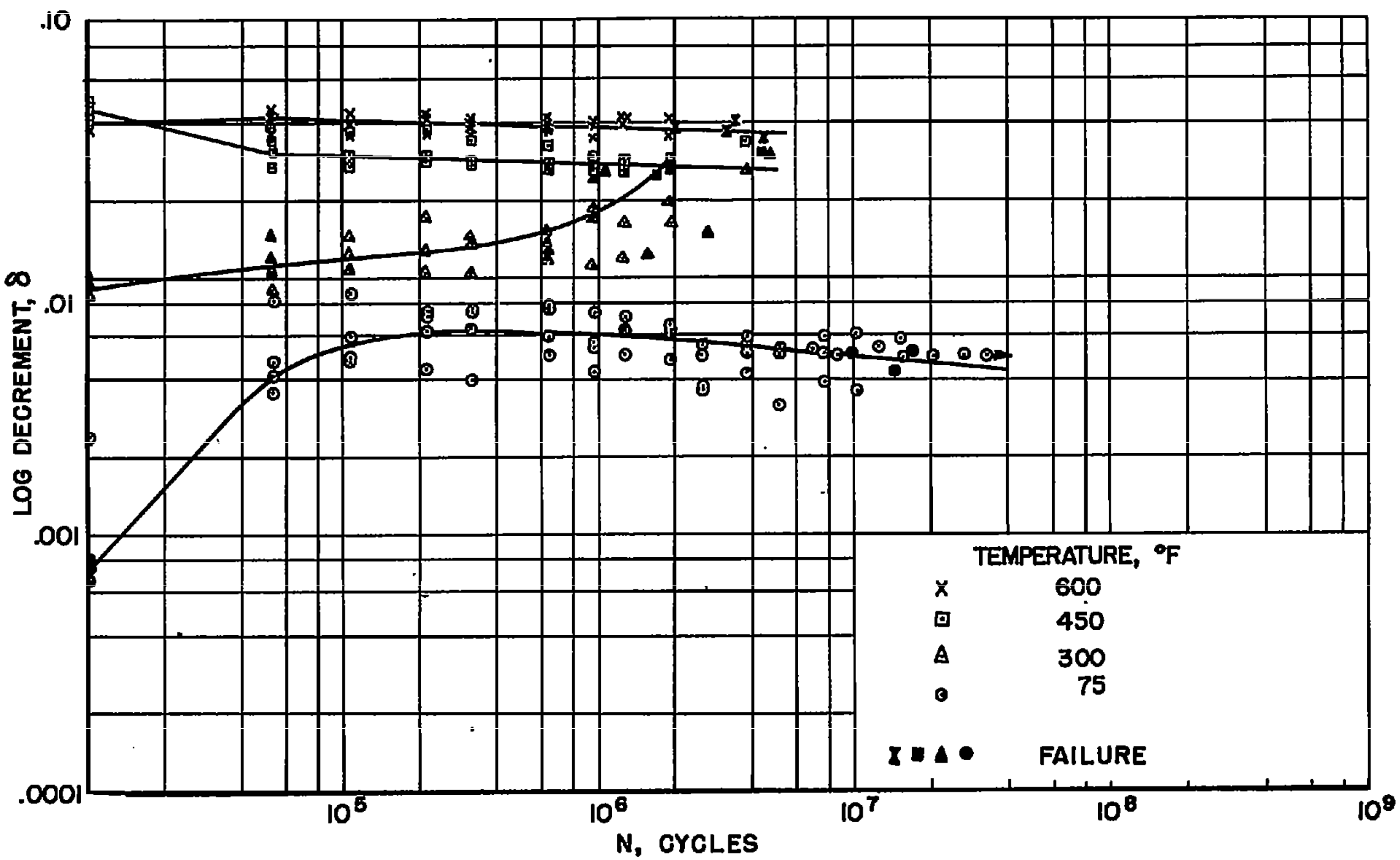
(b) Stress, 3,500 psi.

Figure 9.- Continued.



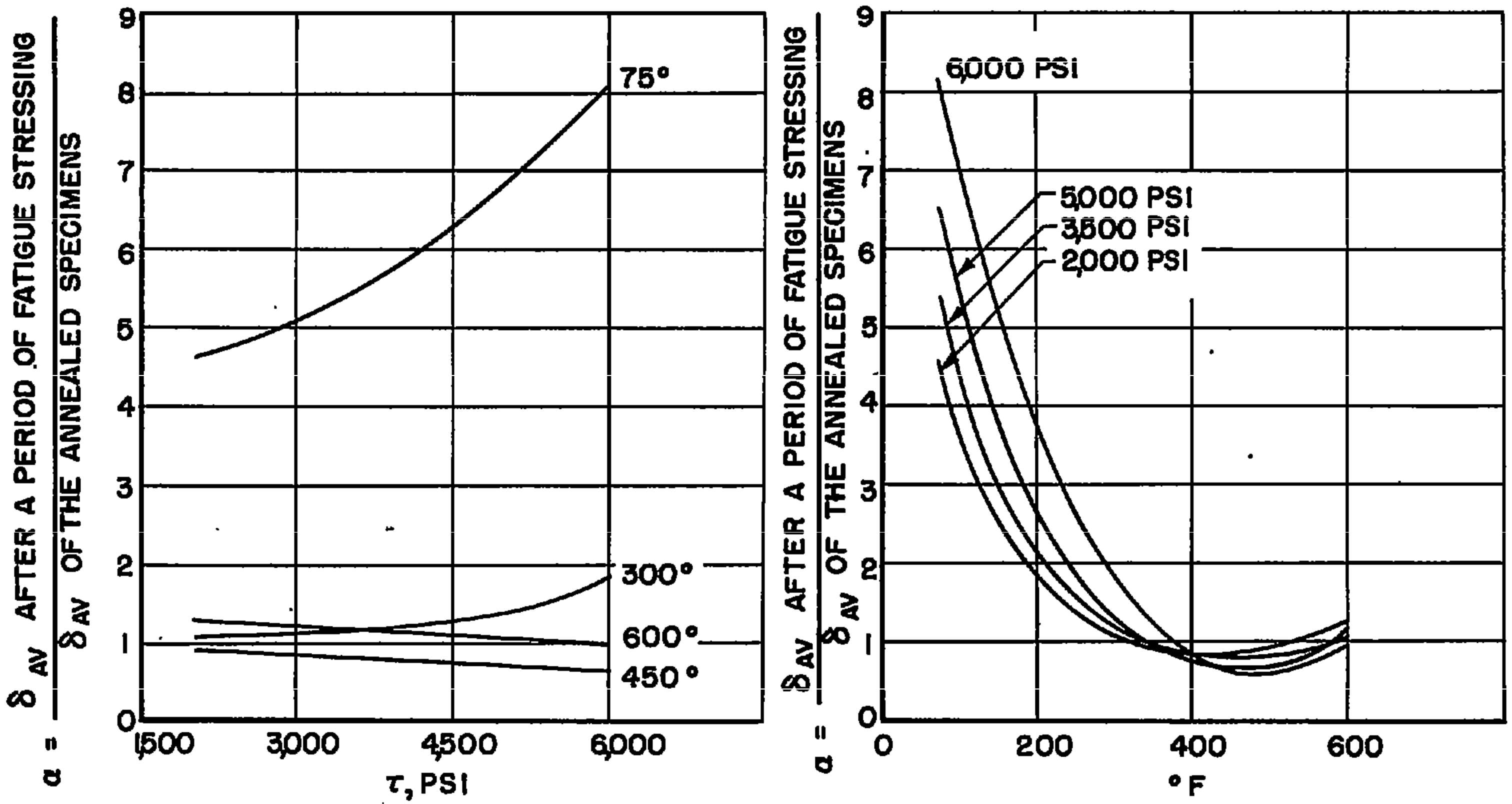
(c) Stress, 5,000 psi.

Figure 9.- Continued.



(d) Stress, 6,000 psi.

Figure 9.- Concluded.



(a) With stress.

(b) With temperature.

Figure 10.- Relative variation of δ_{AV} due to fatigue stressing.

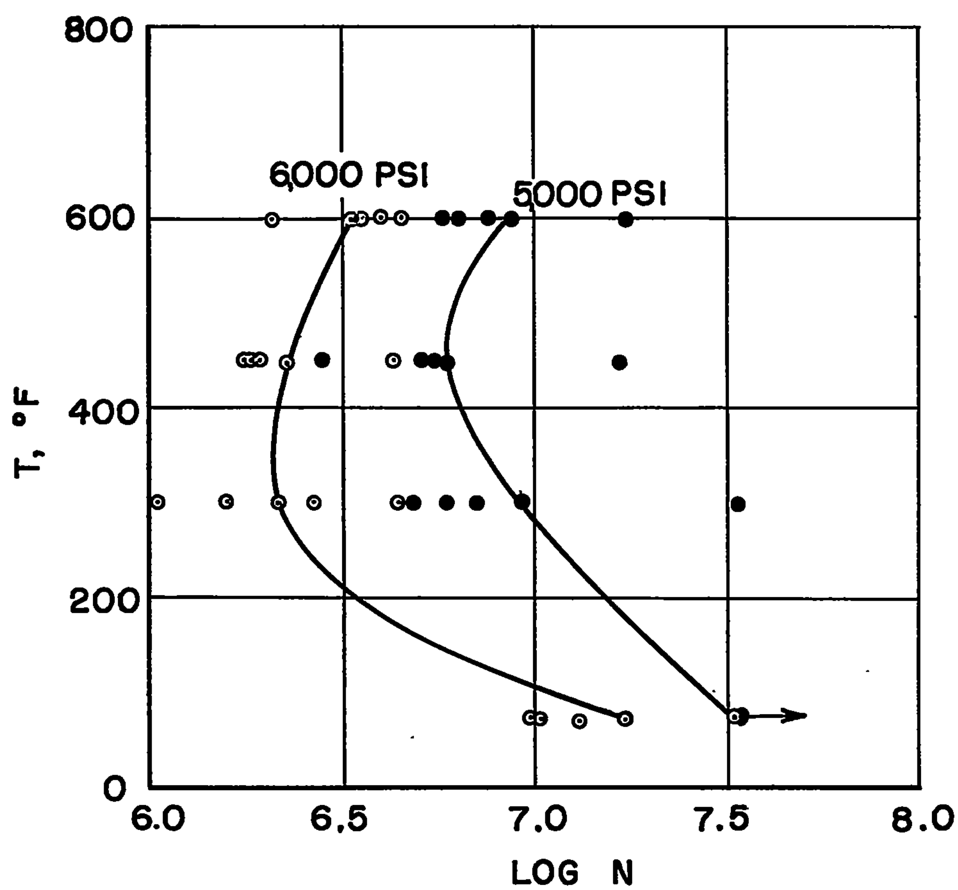
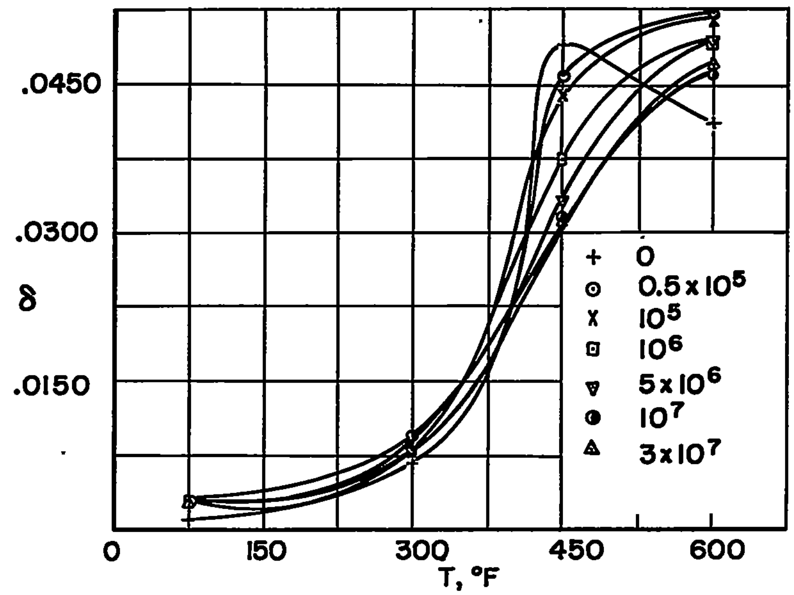
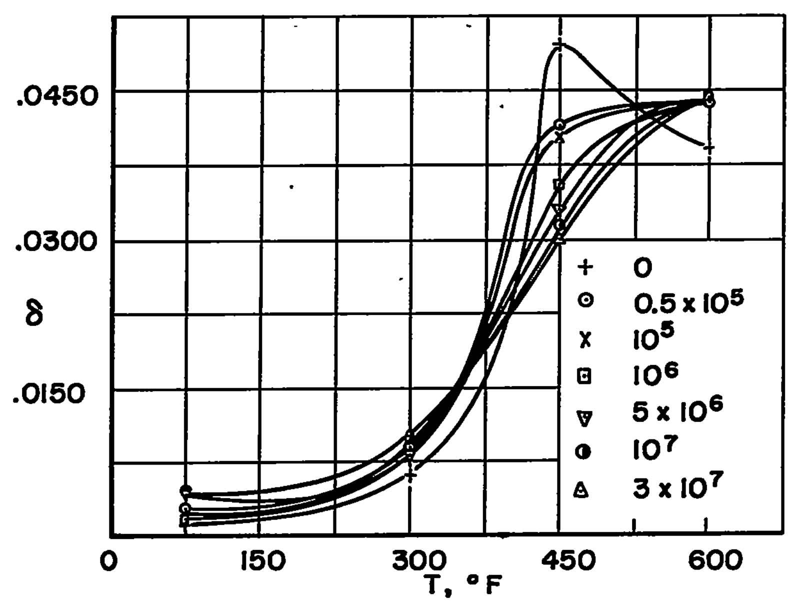


Figure 11.- Variation of fatigue life with temperature.

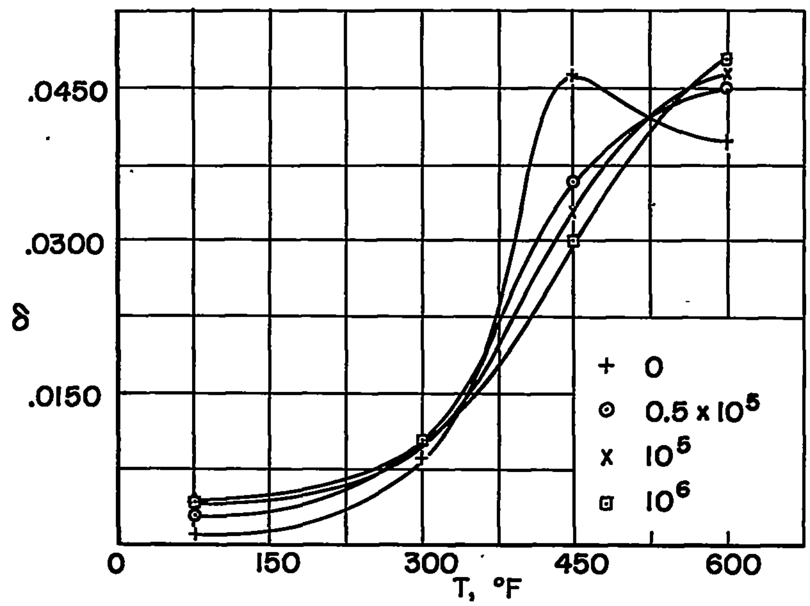


(a) Stress, 2,000 psi.

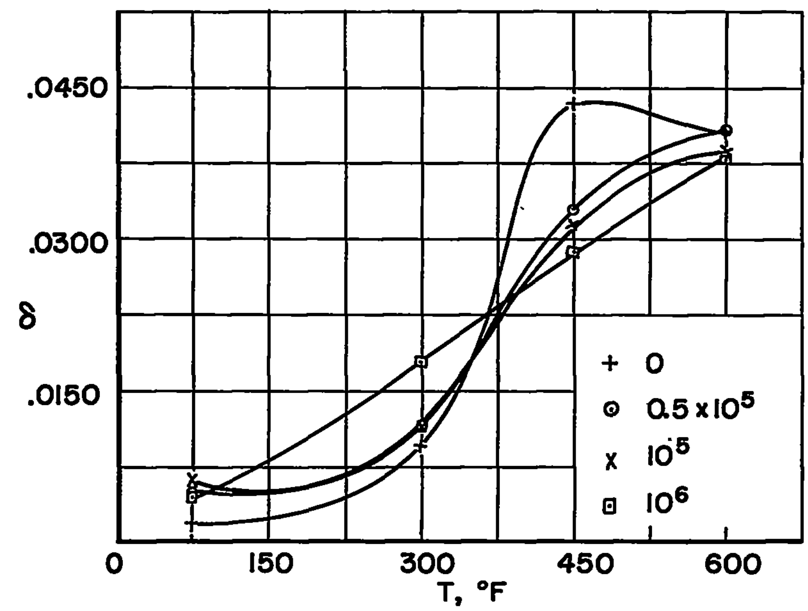


(b) Stress, 3,500 psi.

Figure 12.- Variation of δ with T after application of fatigue stressing.



(c) Stress, 5,000 psi.



(d) Stress, 6,000 psi.

Figure 12.- Concluded.

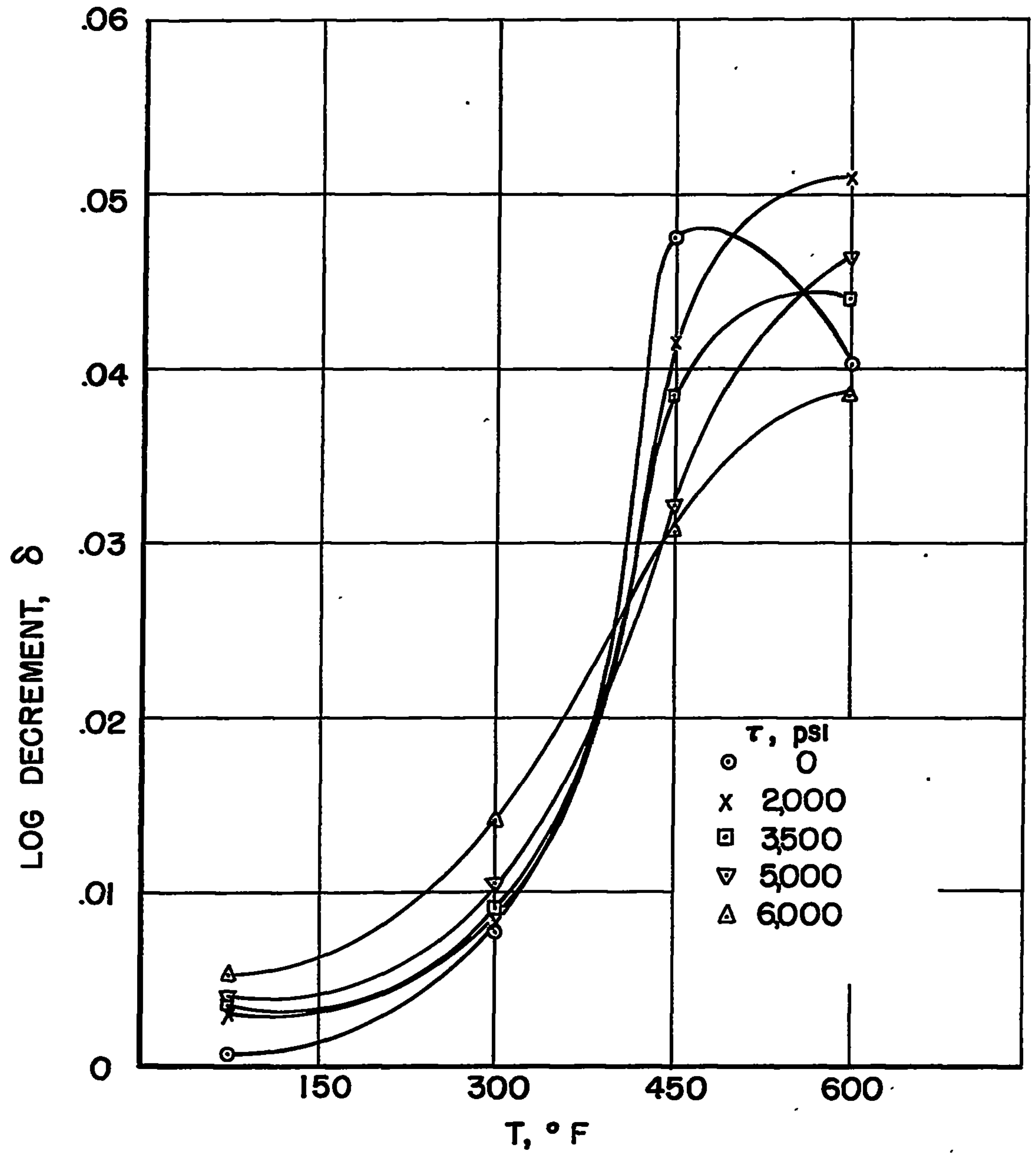


Figure 13.- Variation of δ with temperature for different stress levels.

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